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A NOVEL ALGORITHM TO EVALUATE THE Q-FACTOR OF THE OPTICAL LINKS

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ABSTRACT

Q-factor is a practical simulation parameter to estimate the error probability of data signal in optical communication system since it does not require a long bit sequence as in the direct measurement of bit error rate (BER). Under Gaussian statistic assumptions, the Q-factor can be translated into BER. This paper presents a novel algorithm to evaluate Q-factor of data signal propagating in fiber optic transmission system. The algorithm is simple and can be easily realized in most programming languages with efficient indexed data container and sorting algorithm. With the rotation technique, the algorithm does not require the information of accumulated dispersion to compensate for the time shift introduced by the moving reference frame mismatch in the simulations of wavelength division multiplexing (WDM) systems, and hence greatly reduces the simulation complexity. This new algorithm is used to predict the error free transmission distance of a hybrid amplified WDM system. The result shows a good agreement between the simulation and experiment.

Keywords: Q-factor, optical communication, bit error rate (BER).

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INTRODUCTION

Since an introduction of internet technology during the fundamental transformation of telecommunication definition in the globalization era, the major traffic in communication links has gradually shifted from voice signals into computer data. Due to a technological advancement in data transmission technology, the definition of telecommunication is hence extended to an integration of all media establishing integrated service networks. As a result, a global demand in high speed data transmission is unavoidable. Recently, optical communication has been proved to be the only reasonable choice to meet such a demand (Senior, 1992). Optical fiber can transmit ultra high speed information with extremely low loss over a wide range of wavelengths. By virtue of this outstanding property, fiber optic communication technologies have been adopted in a variety of transmission systems throughout the world, such as international undersea networks and terrestrial links (Kaminow et al., 1997).

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In recent years, considerable efforts have been made to develop effective approaches to increase the transmission rates of an optical link. In general, the maximum transmission rate of a link is limited by losses, dispersions, nonlinearities, and amplified spontaneous emission (ASE) noises within the transmission channels. A number of techniques such as optical amplification, dispersion management, soliton transmission, *etc.*, have been proposed to alleviate the problems (Agrawal, 1995). To fully evaluate the effectiveness of those techniques, a series of experiments must be carried out. Unfortunately, each set-up for an experiment in optical fiber technology is extremely costly and sometimes not feasible. An alternative approach is to exploit some mathematical models to predict the effect of each device within a link. This computer simulation technique is widely used among the link design engineers to evaluate their set-up prior to launching an experiment (Cai et al., 2002).

In optical communication, one of the most important parameter that measures the performance criterion of an optical link is bit error rate (BER), which is defined as the probability of incorrect identification of a bit by the decision circuit at the receiver (Agrawal, 1997). In numerical simulations, however, the BER does not seem to be a practical measure for the system performance as a large number of bits are required to justify the statistical validity. Nonetheless, an estimate of the BER via another statistical value, so called Q-factor, is possible.

The Q-factor calculation is achieved by taking samples of detected signal at the receiver end, when a set of limited length pseudo-random bit sequence (PRBS) is transmitted. The Q-factor is then determined from the statistical properties of detected marks and spaces within the time slot. In practice, the detecting time position within the slot is usually optimized to deliver the best transmission or, in other words, the maximum Q-factor (Agrawal, 1997). In conventional numerical algorithm, this detecting time optimization is carried out by numerically searching through the time slot for the best Q-factor. Unfortunately, this process is not convenient for Wavelength division multiplexing (WDM) simulations as each channel may incur a time shift due to carrier frequency mismatch. As a result, the information about the accumulated dispersion of each channel along the propagation distance must be known at the time of Q-factor evaluation to shift back the signal. This causes a programming complexity issue, when a large scale transmission system simulation including various types of optical devices is concerned.

In this paper, a novel algorithm to compute the Q-factor of an optical data signal received at the receiver end of an optical link is presented. The algorithm uses rotation technique to eliminate the need for accumulated dispersion information as required in the conventional counterpart. The outline of this paper is as follows: First, the probability theory connecting the Q-factor and BER under some certain conditions generally met by optical transmission is established. Next, the proposed algorithm to measure the Q-factor of a set of data signal is described. Finally, the simulation results to confirm the validity of the algorithm are presented.

Q-FACTOR

In optical communications, the performance criterion for digital receiver is governed by bit error rate (BER). For example, a BER of $1\text{x}10^{-9}$ corresponds to an average one error per a billion bits. In numerical simulations, BER is not a practical measure for system performance as an extremely large number of bits are required. An alternative approach is to estimate the BER via Q-factor, which can be easily obtained from a limited number of signal samples. To establish a relation between BER and Q-factor, one may consider a sample set of random signals with noises received at the receiver end.

For a set of received signals at a decision time, the sampled voltage value V fluctuates from bit to bit around the average values V_1 and V_0 for the corresponding "mark" and "space" bits, respectively. The decision circuit compares the sampled value with a decision level V_D and calls it "mark" bit if $V > V_D$ or "space" bit if $V < V_D$.

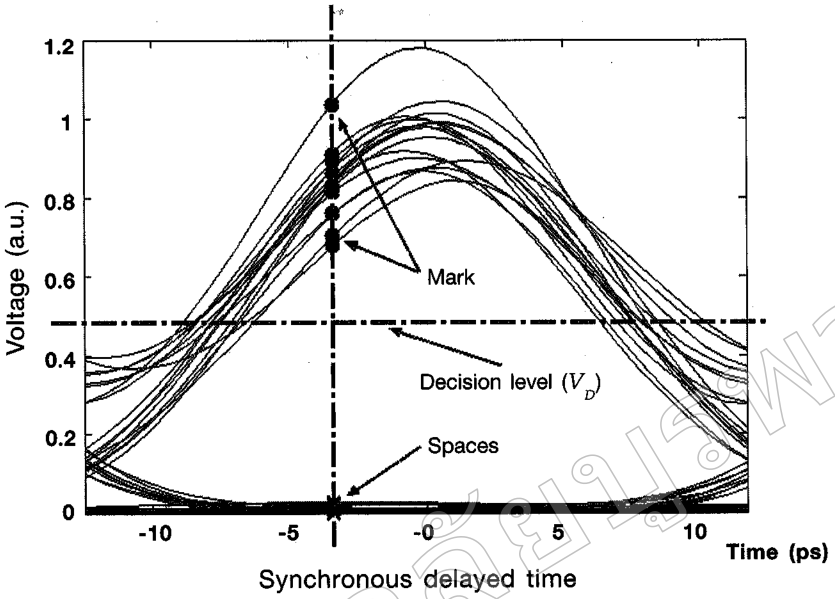

Figure 1. Evaluation of the Q-factor from an eye diagram.

Figure 1 shows the eye diagram of the received signal in the electrical domain after detection process. The dots represent the detected mark bits whereas the crosses are the detected space bits. An error occurs if $V < V_D$ for a mark bit or $V > V_D$ for a space bit due to receiver noise. One may determine the error probability as follows:

$$BER = p(1)P(0/1) + p(0)P(1/0), \tag{1}$$

Where $p(1)$ and $p(0)$ are the probabilities of receiving mark and space bits, respectively. $P(1/0)$ is the probability of deciding space when a mark is received, and $P(1/0)$ is the opposite. If the mark and space bits are equally likely to occur, $p(1) = p(0) = 1/2$, Eq. (1) yields,

$$BER = \frac{1}{2}\left[P(0/1) + P(1/0)\right]. \tag{2}$$

Although, the major noise in optical communication systems is contributed mainly by ASE of the optical amplifiers, which can be categorized into thermal noise class justifying the Gaussian statistic assumption; the Probability Density Function (PDF) of random phenomena affecting transmission performance, such as dispersion fluctuations and nonlinear interactions, have not yet been well understood and might not lend itself to the Gaussian probability. However, at design level, it is practical to assume that those random effects also follow Gaussian statistics (Kurukitkoson, 2004). The error probabilities thus take the forms:

$$P(0/1) = \frac{1}{\sigma_1\sqrt{2\pi}}\int_{-\infty}^{V_D} \exp\left(-\frac{(V-V_1)^2}{2\sigma_1^2}\right)dV = \frac{1}{2}\,\mathrm{erfc}\left(\frac{V_1-V_D}{\sigma_1\sqrt{2}}\right), \tag{3}$$

$$P(1/0) = \frac{1}{\sigma_0\sqrt{2\pi}}\int_{V_D}^{\infty} \exp\left(-\frac{(V-V_0)^2}{2\sigma_0^2}\right)dV = \frac{1}{2}\,\mathrm{erfc}\left(\frac{V_D-V_0}{\sigma_0\sqrt{2}}\right), \tag{4}$$

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where σ_1^2 and σ_0^2 are corresponding variances of the mark and space detected values. V_1 and V_0 correspond to the average voltages of the mark and space detected, respectively. The erfc(·) is the complement error function defined by

$$\mathrm{erfc}(x) = \frac{2}{\pi}\int_x^{\infty} \exp(-y^2)\, dy. \tag{5}$$

By substituting Eqs. (3) and (4) in Eq. (2), the BER yields

$$BER = \frac{1}{4}\left(\mathrm{erfc}\left(\frac{V_1-V_D}{\sigma_1\sqrt{2}}\right)+\mathrm{erfc}\left(-\frac{V_D-V_0}{\sigma_0\sqrt{2}}\right)\right) \tag{6}$$

Eq. (6) suggests that the BER depends on the decision level V_D. In practice, the decision circuit will be tuned to achieve the optimal V_D yielding the best BER. Therefore, by minimizing the BER, the optimal V_D is chosen such that

$$\frac{(V_D-V_0{}^2)}{2\sigma_0^2} = \frac{(V_1-V_D)^2}{2\sigma_1^2} + \mathrm{In}\left(\frac{\sigma_1}{\sigma_0}\right). \tag{7}$$

In most cases of practical interest, the last term of the above equation can be neglected (Agrawal, 1995) resulting in a simple form of equation from which the optimal V_D can be obtained as such

$$\frac{(V_D-V_0)}{\sigma_0} = \frac{(V_1-V_D)}{\sigma_1} = Q \tag{8}$$

Therefore, the optimal V_D yielding the best BER is given by

$$V_D = \frac{\sigma_0 V_D + \sigma_1 V_0}{\sigma_0 + \sigma_1} \tag{9}$$

The Q-factor can then be obtained from

$$Q = \frac{V_1 - V_0}{\sigma_1 + \sigma_0} \tag{10}$$

Finally, the BER of the signal at the receiver with optimal decision level can be obtained from the Q-factor as follows:

$$\mathrm{BER} = \frac{1}{4}\ \mathrm{erfc}\left(\frac{Q}{\sqrt{2}}\right) \tag{11}$$

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One may see that Q-factor evaluation requires only average statistical properties of the detected mark and space bits. In this case, only a few hundred sample bits are sufficient to yield an accurate estimation of Q-factor. Consequently, it is generally more convenient in numerical simulations to use the Q-factor as a measure of system performance, because there is no need to run the simulations with too long random signal sequences until the BER can be measured.

ALGORITHM

In accordance with equation (10), the Q-factor can be calculated from the mean and variance values of the detected marks and spaces at the best synchronous delayed time giving maximum Q. Unlike real transmission systems, the information of bit sequence pattern is generally available at the time of Q-factor calculation in computer simulation. It is thus easy to extract the voltage levels of mark and space bits, and hence implies the optimal decision level. The procedure becomes more complicated, if the simulation involves wavelength division multiplexing (WDM) transmission, in which the pulses at each channel are transmitted at different carrier frequencies. This results in a different shift in time for each channel. In conventional algorithm, the accumulated dispersion for each channel at the evaluation point must be known to shift back the signal prior to Q-factor calculation. This introduces a programming complexity especially in a large scale optical transmission system simulation comprising with various types of optical devices. In this newly proposed algorithm, such information is not necessary as the shifting process is included in the form of bit pattern rotation instead.

Moreover, the use of the rotation technique allows a reconstruction of the bit sequence to be compared with the original to avoid a trivial Q value, should the wrong bit sequence was detected in the first place. This will help improve the correctness of the Q-factor evaluation. Similar to the conventional one, the algorithm involves a search for the best Q value from all samples in a bit slot. For each time instant in the bit slot of all bits, the following procedure is to perform:

1. Record the signal amplitude and its associated bit number. For example, the first bit has the detected amplitude for this time instant of 1.2 V, thus the record should contain (1.2, 1), where the first and the second members are signal amplitude and bit number, respectively.

2. Perform an amplitude sort of the obtained records in ascending order. Figure 2 illustrates the example of the record after sorting algorithm has been applied. In this paper, the ANSI C++ *map* standard container and its efficient built-in sorting algorithm (Stroustrup, 1997) are employed.

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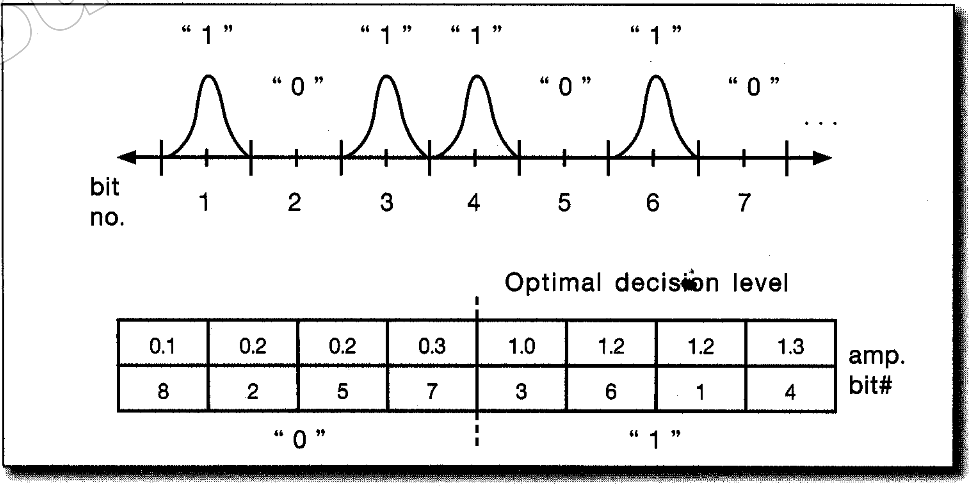

0.1	0.2	0.2	0.3	1.0	1.2	1.2	1.3
8	2	5	7	3	6	1	4

Figure 2. Example of data record after a sorting algorithm has been applied.

3. As we know the number of "mark" and "space" bits, the position of optimal decision level is known instantly. The optimal decision level is represented by the dashed line in Figure 2. Therefore, all records on the left hand side of the decision level are regarded and treated as the "space" bits and vice versa for the "mark" bits.

4. Reconstruct a string of bit sequence from the recorded signals. For instance, a 8-bit sequence from the example in Figure 2 can be reconstructed as "10110100".

5. Compare the string to the original sequence, rotate the sequence as necessary. If the match is found, calculate the Q-factor from the amplitude information obtained. If no match is found, the situation should be regarded that the Q-factor is incomputable and Q = 0 should be returned.

SIMULATION RESULTS

To demonstrate the use of the proposed algorithm, a direct simulation of data transmission through a hybrid amplification WDM transmission system as discussed in Ania-Castanon et al. (2004) has been performed. Figure 3 shows the system schematic including a period of optical scheme. The optical source is a 4-channel WDM transmitter with 100 GHz channel spacing centered at 1.55 m. The signal is represented by a pseudo-random bit sequence (PRBS) of 256 bit length per channel. The pulse format is return-to-zero (RZ) Gaussian with 12.5 ps full-width at half-maximum (FWHM) pulse width. Each span of the optical scheme is comprised with 100 km of single mode fiber (SMF) and 17 km of dispersion compensating fiber (DCF). The hybrid amplification system consists of a backward pumped distributed Raman amplifier (DRA) and an Erbium doped fiber amplifier (EDFA) with a noise figure of 4.5 dB. In this simulation, the amplification ratio () has been set to 0.55 yielding the optimal configuration (Ania-Castanon et al., 2004).

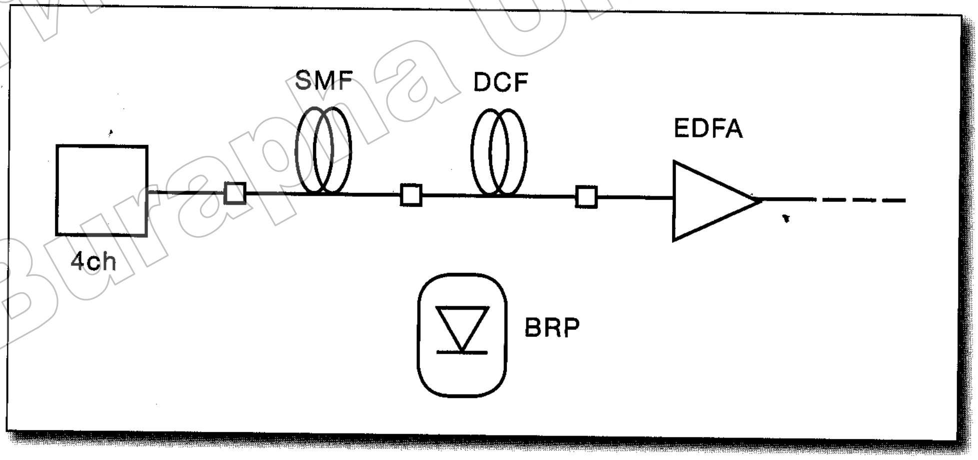

Figure 3. Hybrid amplification system schematic.

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The signal transmission within the fiber has been calculated by numerically solving the well-known Nonlinear Schrodinger Equation using split-step Fourier method (Agrawal, 1995). The Q-factor of each channel is evaluated at each end of every period. Figure 4 illustrates the Q-factor evolution of each channel over a transmission distance of 2250 km. The optical WDM signal is demultiplexed by a super-Gaussian optical filter with a bandwidth of 80 GHz, and detected by a square law detector. The detected electrical signal is then filtered by a low pass 5th order Bessel filter with the bandwidth of 42 GHz before feeding to the Q-factor evaluator. The result shows that the Q-factor exponentially decreases with transmission distance and reaches the value of 6 at the distance about 1400 km. Refer to Eq. (11), this Q-factor value corresponds to a BER of 10^{-9}, which is the maximum error rate acceptable for error-free transmissions. The eye diagram of received signals in channel 2 signal at the error-free transmission limit is also shown in Figure 4. The simulation result is in a good agreement with the experimental data presented by Ania-Castanon et al. (2004), which marks the validity of the algorithm.

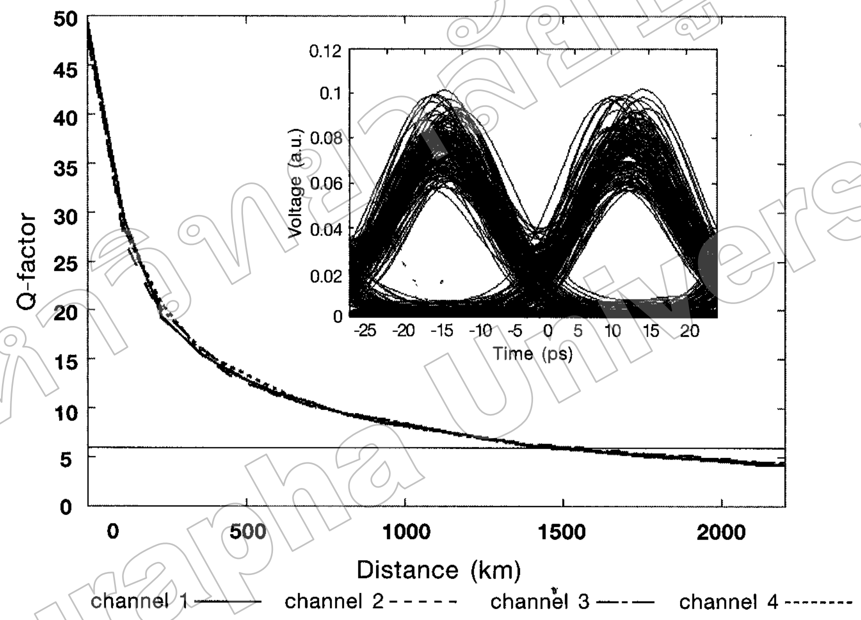

Figure 4. Q-factor evolution of each channel. The inset figure shows the eye diagram of channel 2 at the maximum error free transmission distance.

CONCLUSIONS

In this paper, a novel algorithm to evaluate the Q-factor of signals in optical transmission system has been proposed. The algorithm is simple and practical to be realized in most programming languages with indexed data container and efficient sorting algorithm. With rotation technique, the algorithm does not require information of accumulated dispersion to compensate for the time shift when used in WDM system simulation resulting in a reduction of programming complexity. To verify its validity, the algorithm was then exploited to measure the Q-factor of signals propagated in a 4-channel WDM hybrid amplified transmission system. The simulation result shows a good agreement between the theoretically predicted error-free transmission distance and the experiments presented in literature confirming the validity of the algorithm.

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