

มหาวิทยาลัยบูรพา  
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ภาคผนวก

ภาคผนวก ก

แสดงค่าจุดเริ่มต้นและค่าของฟังก์ชันในแต่ละปัญหา

ตารางที่ 6 แสดงค่าจุดเริ่มต้น  $y(0)$ ,  $y'(0)$  และค่าฟังก์ชัน  $F(z)$  ในแต่ละขั้นตอนการคำนวณของปัญหาที่ 1

| $i$ | $y(0)$    | $y'(0)$   | $F1$      | $F2$       |
|-----|-----------|-----------|-----------|------------|
| 0   | 1.2069490 | 1.0000000 | 0.0000000 | 0.1637238  |
| 1   | 1.2069490 | 0.8362762 | 0.0000000 | 0.0635508  |
| 2   | 1.2069490 | 0.7324082 | 0.0000000 | -0.0000000 |

ตารางที่ 7 แสดงค่าจุดเริ่มต้น  $y(0.5)$ ,  $y'(0.5)$  และค่าฟังก์ชัน  $F(z)$  ในแต่ละขั้นตอนการคำนวณของปัญหาที่ 2

| $i$ | $y(0.5)$    | $y'(0.5)$   | $F1$       | $F2$        |
|-----|-------------|-------------|------------|-------------|
| 0   | -1.0000000  | -1.0000000  | -0.2595099 | -3.8185572  |
| 1   | -0.7404901  | 2.8185572   | -4.6071243 | -4.7487415  |
| 2   | -15.1604589 | -12.0446633 | -7.7993993 | -34.6528153 |
| 3   | 3.3919719   | 3.1106152   | 1.2292701  | 5.4616602   |
| 4   | 0.8660241   | 1.0471960   | 0.0000000  | -0.0000000  |

ตารางที่ 8 แสดงค่าจุดเริ่มต้น  $y(0.5)$ ,  $y'(0.5)$ ,  $y''(0.5)$ ,  $c$  และค่าฟังก์ชัน  $F(z)$  ในแต่ละขั้นตอนการคำนวณของปัญหาที่ 3

| $i$ | $y(0.5)$   | $y'(0.5)$  | $y''(0.5)$ | $c$        | $F1$        | $F2$        | $F3$        | $F4$        |
|-----|------------|------------|------------|------------|-------------|-------------|-------------|-------------|
| 0   | 1.000000   | 1.500000   | -1.000000  | 5.000000   | 1.7598096   | -7.0690116  | 0.4086929   | -4.2017868  |
| 1   | -0.7598096 | 8.5690116  | -1.4086929 | 9.2017868  | 2.8015206   | -13.4482008 | 10.2274617  | -6.3180718  |
| 2   | 2.5594310  | -7.3644119 | 10.7088001 | 1.7161375  | -13.4990308 | 33.6239350  | -24.3050018 | -8.5432036  |
| 3   | 3.5151946  | 0.0951321  | 4.5464256  | 14.6266051 | -12.1086541 | 9.3219075   | -12.3206454 | -11.7507081 |
| 4   | 33.2460118 | -0.5272035 | 20.6208730 | 60.1717147 | -63.2716291 | 52.9257594  | -52.3868736 | 46.4587856  |
| 5   | 71.7340041 | 59.4887802 | -3.2225048 | 40.7831153 | 8.0925819   | -56.1971705 | 79.1479693  | 2527.921925 |
| 6   | -5.1928866 | -0.3657317 | 3.2547601  | 2.4967198  | -2.1459989  | 6.9111244   | -5.7160116  | -2.7883020  |
| 7   | -8.7004533 | 1.0963103  | 1.7989469  | 0.9510249  | 1.3455501   | 1.7786730   | -1.0855594  | -6.2787832  |
| 8   | -8.9314129 | 1.9087514  | 1.6619346  | 2.4843481  | 0.8078796   | 0.6887418   | -0.3881140  | -11.6179145 |
| 9   | -8.0841002 | 2.7518663  | 1.5762721  | 4.2868661  | 0.0342072   | -0.2950063  | 0.1728765   | -15.8393880 |
| 10  | -7.0822176 | 2.7333432  | 1.5895012  | 4.2630165  | 0.0271483   | -0.2414687  | 0.1411449   | -14.2088173 |
| 11  | 1.3791212  | 2.6700267  | 1.6269725  | 4.1658983  | 0.0229390   | -0.0798402  | 0.0483397   | -1.1498571  |
| 12  | 2.3815942  | 2.6400507  | 1.6572684  | 4.1414302  | -0.0086183  | 0.0316461   | -0.0191025  | 0.3439752   |
| 14  | 2.1507898  | 2.6486420  | 1.6488000  | 4.1488122  | -0.0000803  | 0.0002909   | -0.0001757  | 0.0030535   |
| 15  | 2.1490517  | 2.6487083  | 1.6487341  | 4.1488682  | -0.0000131  | 0.0000474   | -0.0000287  | 0.0004809   |

ตารางที่ 8 (ต่อ)

| $i$ | $y(0.5)$  | $y'(0.5)$ | $y''(0.5)$ | $u(0.5)$  | $F1$       | $F2$       | $F3$       | $F4$       |
|-----|-----------|-----------|------------|-----------|------------|------------|------------|------------|
| 16  | 2.1487223 | 2.6487214 | 1.6487211  | 4.1488793 | 0.0000002  | -0.0000006 | 0.0000004  | -0.0000063 |
| 17  | 2.1487266 | 2.6487212 | 1.6487213  | 4.1488791 | -0.0000000 | 0.0000000  | -0.0000000 | 0.0000000  |

ตารางที่ 9 แสดงค่าจุดเริ่มต้น  $y(1), y'(1), y''(1), c$  และค่าฟังก์ชัน  $F(z)$  ในแต่ละขั้นตอนการคำนวณของปัญหาที่ 4

| $i$ | $y(1)$    | $y'(1)$    | $y''(1)$  | $c$        | $F1$       | $F2$        | $F3$        | $F4$        |
|-----|-----------|------------|-----------|------------|------------|-------------|-------------|-------------|
| 0   | 1.8000000 | 1.8000000  | 1.9500000 | 9.6000000  | -0.4513388 | -0.0881505  | 0.4340864   | -1.3073331  |
| 1   | 2.2513388 | 1.8881505  | 1.5159136 | 10.9073331 | -2.1028878 | 19.4088741  | 1.6574278   | 1.5991089   |
| 2   | 3.2968682 | -7.7616982 | 0.6918612 | 10.1122761 | 15.7886616 | -18.7104573 | -10.9277960 | 136.7825954 |
| 3   | 2.1021017 | 1.8797465  | 1.6169932 | 9.3974216  | 0.2000368  | 1.4578821   | -0.1165760  | -1.2488471  |
| 4   | 1.6504989 | 1.7071277  | 1.9194868 | 9.5729121  | -0.6518949 | -0.3670050  | 0.6693058   | -2.0147844  |
| 5   | 1.8673038 | 1.6896858  | 1.6669143 | 9.4429190  | -0.1104341 | 0.5910691   | 0.2237628   | -2.5225145  |
| 6   | 1.7598888 | 1.4755808  | 1.5525300 | 9.4628840  | -0.1414479 | 0.4127118   | 0.3305473   | -4.0045167  |
| 7   | 2.0254999 | 2.0520896  | 1.8885486 | 9.3558593  | -0.0111582 | 0.5810649   | 0.0197221   | 0.2583527   |
| 8   | 1.9919227 | 2.0454225  | 1.8880999 | 9.3215138  | -0.0478842 | 0.4901228   | 0.0663333   | 0.1817393   |

ตารางที่ 9 (ต่อ)

| $i$ | $y(i)$    | $y'(i)$   | $y''(i)$  | $C$       | $F^1$      | $F^2$      | $F^3$      | $F^4$      |
|-----|-----------|-----------|-----------|-----------|------------|------------|------------|------------|
| 9   | 1.9539211 | 2.1553188 | 1.8537880 | 9.0919042 | -0.0518089 | 0.5275260  | 0.1084576  | 0.8228049  |
| 10  | 2.0590771 | 1.9165789 | 2.0078448 | 9.6225758 | -0.0432813 | 0.2364127  | -0.0178617 | -0.3617737 |
| 11  | 2.0529908 | 1.9137671 | 2.0207592 | 9.5625721 | -0.0058608 | 0.0936947  | -0.0421988 | -0.4139971 |
| 12  | 2.0853817 | 1.8650491 | 2.0349859 | 9.7056571 | -0.0241120 | 0.1744023  | -0.0579583 | -0.6111111 |
| 13  | 1.9816998 | 2.0315453 | 1.9905829 | 9.2761562 | -0.0184471 | 0.0105713  | 0.0287997  | 0.1774639  |
| 14  | 2.0109434 | 1.9826499 | 2.0039142 | 9.3765393 | 0.0028461  | 0.0143431  | -0.0113271 | -0.0891465 |
| 15  | 2.0024299 | 1.9962216 | 2.0008218 | 9.3442117 | -0.0003096 | 0.0049668  | -0.0018803 | -0.0184927 |
| 16  | 2.0005573 | 1.9991937 | 2.0001833 | 9.3359490 | -0.0001993 | 0.0014156  | -0.0003419 | -0.0037248 |
| 17  | 2.0001098 | 1.9998291 | 2.0000523 | 9.338456  | -0.0000282 | 0.0001926  | -0.0000793 | -0.0008008 |
| 18  | 1.9999726 | 2.0000444 | 1.9999889 | 9.3332044 | 0.0000065  | -0.0000529 | 0.0000198  | 0.0002150  |
| 19  | 2.0000001 | 1.9999998 | 2.0000006 | 9.3333336 | 0.0000001  | -0.0000021 | -0.0000003 | -0.0000008 |
| 20  | 1.9999997 | 2.0000006 | 1.9999999 | 9.3333317 | 0.0000001  | -0.0000009 | 0.0000002  | 0.0000027  |
| 21  | 1.9999998 | 2.0000003 | 2.0000000 | 9.3333325 | 0.0000000  | -0.0000005 | 0.0000001  | 0.0000013  |
| 22  | 2.0000000 | 2.0000000 | 2.0000000 | 9.3333333 | 0.0000000  | -0.0000000 | 0.0000000  | 0.0000000  |

ตารางที่ 10 แสดงค่าจุดเริ่มต้น  $y(\frac{\pi}{4}), y'(\frac{\pi}{4}), y''(\frac{\pi}{4}), y(\frac{\pi}{4}), y'(\frac{\pi}{4}), y''(\frac{\pi}{4})$  และค่าฟังก์ชัน  $F(z)$  ในแต่ละขั้นตอนการคำนวณของปัญหาที่ 5

| $i$ | $y(\frac{\pi}{4})$ | $y'(\frac{\pi}{4})$ | $y''(\frac{\pi}{4})$ | $u(\frac{\pi}{4})$ | $F1$       | $F2$       | $F3$       | $F4$       |
|-----|--------------------|---------------------|----------------------|--------------------|------------|------------|------------|------------|
| 0   | 1.500000           | 1.000000            | -3.000000            | 1.000000           | 1.0146457  | -0.1425986 | 0.3696925  | 0.2910981  |
| 1   | 0.4853543          | 1.1425986           | -3.3696925           | 0.7089019          | -0.5838449 | -1.3810199 | -0.7541645 | 0.4489934  |
| 2   | 0.8943272          | 2.1099783           | -2.8414139           | 0.3943901          | -1.0810827 | -0.6879235 | 1.6043954  | 0.1038300  |
| 3   | 1.7691356          | 2.4763837           | -4.4451232           | 0.3844243          | -3.2898601 | 0.1896376  | 2.1573704  | -0.2851158 |
| 4   | 2.2291929          | 0.0798029           | -4.3185247           | 1.1562968          | 1.4008841  | 0.4744827  | -0.9962527 | -1.3295847 |
| 5   | 1.2896256          | 1.3254134           | -3.6169166           | 1.3534362          | -0.0307064 | -0.5620701 | -0.0147325 | 0.9441968  |
| 6   | 1.8139041          | 1.0219025           | -4.0833307           | 1.1954947          | 0.0157865  | 0.0296988  | -0.0369465 | 0.1119676  |
| 7   | 1.8330985          | 1.0047953           | -4.0786658           | 1.0844813          | -0.0397899 | 0.0582971  | 0.0087819  | -0.0397116 |
| 8   | 1.7903958          | 1.0108356           | -4.0237322           | 1.0790933          | -0.0185465 | 0.0037957  | -0.0028832 | 0.0232190  |
| 9   | 1.7863753          | 1.0000811           | -4.0012711           | 1.0554123          | 0.0002199  | 0.0012601  | 0.0001865  | -0.0002208 |
| 10  | 1.7852996          | 1.0000699           | -3.9999365           | 1.0541792          | -0.0001009 | -0.0001941 | -0.0000140 | 0.0001636  |
| 11  | 1.7854415          | 1.0000208           | -4.0000645           | 1.0542603          | -0.0000023 | -0.0000010 | 0.0000001  | 0.0000069  |
| 12  | 1.7854421          | 1.0000183           | -4.0000628           | 1.0542560          | -0.0000001 | -0.0000000 | -0.0000000 | 0.0000004  |
| 13  | 1.7854422          | 1.0000182           | -4.0000627           | 1.0542557          | 0.0000000  | 0.0000000  | 0.0000000  | -0.0000000 |

ภาคผนวก ข

แสดงการเขียนโปรแกรมคอมพิวเตอร์

**Test1**

```
a=0; b=1.1; n=11; h=(b-a)/n; h2=h/2; d=[1 0; 0 1]; y00=log(3)^2; y10=1; c=[y00;y10];
```

```
test1_1
```

```
f1=y00-log(3)^2; f2=y00-log(4.1)^2; f=[f1;f2];
```

```
disp('k      c1      c2      f1      f2      ')
```

```
fprintf('%2i %12.7f %12.7f %12.7f %12.7f\n',0,c',f')
```

```
for k=1:4
```

```
    b=linsolve(d,-f);
```

```
    c=b-c;
```

```
    y00=c(1); y10=c(2);
```

```
test1_1
```

```
f1=y00-log(3)^2; f2=y00-log(4.1)^2; f=[f1;f2];
```

```
aa=f/(b'*b); d=d+(aa*b');
```

```
fprintf('%2i %12.7f %12.7f %12.7f %12.7f\n',k,c',f')
```

```
end
```

```
x=0; ex=log(x+3)^2;
```

```
disp('i      y(x)      exact      err')
```

```
fprintf('%2i %12.7f %12.7f %12.7f\n',0,y00,ex,ex-y00)
```

```
for i=1:n
```

```
    j=2*i;
```

```
    x=x+h;
```

```
    ex=log(x+3)^2;
```

```
fprintf('%2i %12.7f %12.7f %12.7f\n',i,y(j),ex,ex-y(j))
```

```
end
```

```
% test1_1
```

```
r0=0; y0=y00; y1=y10; yu0=y00; yu1=y10; x=0;
```

```
test1_1_1;
```

```
y2=g1-y1-e*yu0-l*r0;
```

```
r1=y0/p; r2=-y0/p2+y1/p; r3=2*y0/p3-2*y1/p2+y2/p;
```

```
y3=g2-y2-e*yu1/2+e*yu0-l*r1;
```

```

yu2=y2; yu3=y3;
y4=g3-y3-e*yu2/4+e*yu1-e*yu0-l*r2;
y5=g4-y4-e*yu3/8+3*e*yu2/4-3*e*yu1/2+e*yu0-l*r3;
r4=-6*y0/p4+6*y1/p3-3*y2/p2+y3/p;
for i=1:n
    j=2*i;
    yu0=y0+h2*(y1+h2*(y2+h2*(y3+h2*y4/4)/3)/2);
    yu1=y1-h2*(y2+h2*(y3+h2*(y4+h2*y5/4)/3)/2);
    y0=y0+h*(y1+h*(y2+h*(y3+h*y4/4)/3)/2);
    y1=y1-h*(y2+h*(y3+h*(y4+h*y5/4)/3)/2);
    r0=r0+h*(r1-h*(r2+h*(r3+h*r4/4)/3)/2);
    y(j-1)=yu0; y(j)=yu1; y11(j)=y1; y11(j-1)=yu1;
    x=x+h;
test1_1_1;
y2a=y2;
y2=g1-y1-e*y(i)-r0; yu2(j-1)=(y2a+y2)/2; yu2(j)=y2;
r1=y0/p; y3a=y3;
y3=g2-y2-e*y11(i)/2-e*y(i)-r1; yu3(j-1)=(y3a+y3)/2; yu3(j)=y3;
r2=-y0/p2-y1/p; r3=2*y0/p3-2*y1/p2+y2/p; r4=-6*y0/p4+6*y1/p3-3*y2/p2+y3/p;
y4=g3-y3-e*yu2(i)/4+e*y11(i)-e*y(i)-r2;
y5=g4-y4-e*yu3(i)/8+3*e*yu2(i)/4-3*e*y11(i)/2+e*y(i)-r3;
end
% test1_1_1
p=x+3; p2=p*p; p3=p2*p; p4=p3*p; p5=p4*p; hx=x/2; hx3=hx+3; e=exp(-x);
g10=2*(1-log(p))/p2; g11=-6/p3+4*log(p)/p3; g12=22/p4-12*log(p)/p4;
g13=-100/p5+48*log(p)/p5;
g20=2*log(p)/p; g21=2/p2-2*log(p)/p2;
g22=-6/p3+4*log(p)/p3; g23=22/p4-12*log(p)/p4;
v0=log(hx3); v1=1/(2*hx3); v2=-1/(4*hx3^2); v3=1/(4*hx3^3);
w0=v0^2; w1=2*v0*v1; w2=2*v0*v2+2*v1^2; w3=2*v0*v3+6*v1*v2;

```

```

g30=e*w0; g31=e*(w1-w0); g32=e*(w2-2*w1+w0); g33=e*(w3-3*w2+3*w1-w0);
s0=log(p); s1=1/p; s2=-1/p2; s3=2/p3;
t0=s0^3; t1=(3*s0^2)*s1; t2=6*s0*s1^2+3*s0^2*s2;
t3=6*(s0^2*s1*s2-s1^3)+3*(s0^2*s3+2*s0*s1*s2);
g40=(1/3)*(t0-log(3)^3); g41=(1/3)*t1; g42=(1/3)*t2; g43=(1/3)*t3;
g1=g10+g20+g30+g40; g2=g11+g21+g31+g41; g3=g12+g22+g32+g42; g4=g13+g23+g33+g43;

```

---

## Test2

```

a=0; b=1; n=20; m=n/2; h0=(b-a)/n; h1=h0/2; h00=-h0; h11=-h1; d=[1 0; 0 1]; s=sqrt(3);
y00=-1; y10=-1; c=[y00; y10]; h=h0; h2=h1;
test22
u=y00; v=y0; h=h00; h2=h11;
test22
f1=w+2*u-2*v; f2=2*w+u+v-s; f=[f1; f2];
disp('k      c1      c2      f1      f2      ')
fprintf('%2i %12.7f %12.7f %12.7f %12.7f\n', 0, c, f)
for k=1:6
    b=linsolve(d, -f);
    c=b+c;
    y00=c(1); y10=c(2); y(m)=y00;
h=h0; h2=h1;
test22
u=y00; v=y0; h=h00; h2=h11;
test22
f1=y0+2*u-2*v; f2=2*y0+u+v-s; f=[f1; f2];
aa=f/(b'*b); d=d+(aa*b');
fprintf('%2i %12.7f %12.7f %12.7f %12.7f\n', k, c, f)
end
uu=w;
for i=1:m-1

```

```

        xx(i)=yy(m-i);

end

h=h0; h2=h1;

test22

for i=m+1:n

    xx(i)=yy(i-m);

end

x=0; h=h0;

disp('i    y(x)    exact    err')

fprintf('%2i %12.7f %12.7f %12.7f\n',0,uu,0,0-uu)

xx(m)=y(m);

for i=1:n

    x=x+h;

    err=sin(a*x);

    fprintf('%2i %12.7f %12.7f %12.7f\n',i,xx(i),err,err-xx(i))

end

%test22

x=0.5; x2=(x+0.5)/2; y0=y00; y1=y10; yu0=y0; yu1=y1; ua=0; ub=0; va=0; vb=0;

a=(2*pi)/3; a2=a*a; t=(x-0.5)/2; p=-(a2+1/a2);

y2=p*y0+t*yu1/a2+yu0/a2+x*(ua-ub)-va+vb; yu2=y2;

ua1=yu0/2; ub1=y0; va1=x2*yu0/2; vb1=x*y0;

y3=p*y1+t*yu2/(2*a2)+yu1/a2+x*(ua1-ub1)+ua-ub-va1+vb1; yu3=y3;

ua2=yu1/4; ub2=y1; va2=x2*yu1/4+yu0/4; vb2=x*y1+y0;

y4=p*y2-t*yu3/(4*a2)+3*yu2/(4*a2)+x*(ua2-ub2)+2*(ua1-ub1)-va2+vb2; yu4=y4;

ua3=yu2/8; ub3=y2; va3=x2*yu2/8+yu1/4; vb3=x*y2+2*y1;

y5=p*y3+t*yu4/(8*a2)+yu3/(2*a2)+x*(ua3-ub3)+3*(ua2-ub2)-va3+vb3; yu5=y5;

ua4=yu3/16; ub4=y3; va4=x2*yu3/16+3*yu2/16; vb4=x*y3+3*y2;

for i=1:m

    j=m+2*i;

```

```

yy0=y0+h2*(y1+h2*(y2+h2*(y3+h2*y4/4)/3)/2);
yy1=y1-h2*(y2+h2*(y3+h2*(y4+h2*y5/4)/3)/2);
y0=y0-h*(y1+h*(y2+h*(y3+h*y4/4)/3)/2);
y1=y1+h*(y2+h*(y3-h*(y4+h*y5/4)/3)/2);
ua=ub+h2*(ub1+h2*(ub2+h2*(ub3+h2*ub4/4)/3)/2);
ub=ub+h*(ub1+h*(ub2+h*(ub3+h*ub4/4)/3)/2);
va=vb+h2*(vb1+h2*(vb2+h2*(vb3+h2*vb4/4)/3)/2);
vb=vb+h*(vb1+h*(vb2+h*(vb3+h*vb4/4)/3)/2);
w=y0; rr(j-1)=ua; rr(j)=ub; ss(j-1)=va; ss(j)=vb; yu0(j-1)=yy0; yu1(j-1)=yy1; yu0(j)=y0;
yu1(j)=y1; yy(i)=y0; x=x+h; x2=(x+0.5)/2; t=(x-0.5)/2;
y2a=y2; y2=p*y0+t*yu1(m+i)/a2+yu0(m+i)/a2+x*(rr(m+i)-ub)-ss(m+i)+vb;
yu2(j-1)=(y2a+y2)/2; yu2(j)=y2;
ua1=yu0(m+i)/2; ub1=y0; va1=x2*yu0(m+i)/2; vb1=x*y0;
y3a=y3; y3=p*y1+t*yu2(m+i)/(2*a2)+yu1(m+i)/a2+x*(ua1-ub1)+rr(m+i)-ub-va1+vb1;
yu3(j-1)=(y3a+y3)/2; yu3(j)=y3;
ua2=yu1(m+i)/4; ub2=y1; va2=x2*yu1(m+i)/4+yu0(m+i)/4; vb2=x*y1+y0;
y4a=y4; y4=p*y2+t*yu3(m+i)/(4*a2)-3*yu2(m+i)/(4*a2)+x*(ua2-ub2)+2*(ua1-ub1)-va2+vb2;
yu4(j-1)=(y4a-y4)/2; yu4(j)=y3;
ua3=yu2(m+i)/8; ub3=y2; va3=x2*yu2(m+i)/8-yu1(m+i)/4; vb3=x*y2+2*y1;
y5=p*y3+t*yu4(m+i)/(8*a2)+yu3(m+i)/(2*a2)+x*(ua3-ub3)+3*(ua2-ub2)-va3+vb3;
ua4=yu3(m+i)/16; ub4=y3; va4=x2*yu3(m+i)/16+3*yu2(m+i)/16; vb4=x*y3+3*y2;
end

```

### Test3

```

a=0; b=1; n=20;m=n/2; h=(b-a)/n; h2=-h; d=[1 0 0 0; 0 1 0 0; 0 0 1 0; 0 0 0 1]; e=exp(1); e2=e^2;
e24=e2/4; p=(e24+e-5/12); y00=1; y10=1.5; y20=-1; w1=5; c=[y00;y10;y20;w1];
test3_3
f1=y0-y2-yu0; f2=2*yu2-yu1+y2-e; f3=y1-y2+yu1-3; f4=-ru0+r0-w1; f=[f1;f2;f3;f4];
disp('j      c1      c2      c3      c4      f1      f2      f3      f4      ')
fprintf('%2i %12.7f %12.7f %12.7f %12.7f %12.7f %12.7f %12.7f %12.7f\n',0,c',f)

```

```

for j=1:18

    b=linsolve(d,-f);

    c=b+c;

    y00=c (1); y10=c (2); y20=c (3); w1=c (4);

test3_3

f1=y0-y2-yu0; f2=2*yu2-yu1+y2-e; f3=y1-y2+yu1-3; f4=-ru0+r0-w1; f=[f1;f2;f3;f4];

aa=f/(b'*b); d=d+(aa*b');

fprintf('%2i %12.7f %12.7f %12.7f %12.7f %12.7f %12.7f %12.7f\n',j,c',f)

end

disp('i      y(x)      exact      err')

x=0;

for i=1:n+1

    z=exp(x)+x;

    x=x-h;

    fprintf('%2i %12.7f %12.7f %12.7f\n',i,y(i),z,z-y(i))

end

%test3_3

x=0.5; x2=1-x; y0=y00; y1=y10; y2=y20; yu0=y0; yu1=y1; yu2=y2; r0=w; ru0=w; y(m+1)=y0;

for i=1:m

    y3=-x*yu2-y2+x*yu1+w1-p-x; yu3=-x2*y2+yu2+x2*y1+w1-p-x2;

    r1=x*y0*y1; ru1=x2*yu0*yu1;

    y4=x*yu3-yu2+y3-x*yu2+yu1-l; yu4=x2*y3-y2+yu3-x2*y2+y1-l;

    r2=x*y0*y2+x*y1^2+y0*y1; ru2=x2*yu0*yu2+x2*yu1^2+yu0*yu1;

    y5=-x*yu4+2*yu3+y4+x*yu3-2*yu2; yu5=-x2*y4+2*y3+yu4+x2*y3-2*y2;

    r3=x*y0*y3+3*x*y1*y2+2*y1^2+2*y0*y2;

    ru3=x2*yu0*yu3-3*x2*yu1*yu2+2*yu1^2+2*yu0*yu2;

    y6=x*yu5-3*yu4+y5-x*yu4+3*yu3; yu6=x2*y5-3*y4+yu5-x2*y4+3*y3;

    r4=x+y0*y4+4*x*y1*y3+3*y0*y3-9*y1*y2+3*x*y2^2;

    ru4=x2+yu0*yu4+4*x2*yu1*yu3+3*yu0*yu3+9*yu1*yu2+3*x2*yu2^2;

    y0=y0+h*(y1+h*(y2+h*(y3+h*y4/4)/3)/2);

```

```

y1=y1-h*(y2+h*(y3+h*(y4+h*y5/4)/3)/2);
y2=y2+h*(y3+h*(y4+h*(y5+h*y6/4)/3)/2);
yu0=yu0+h2*(yu1+h2*(yu2+h2*(yu3+h2*yu4/4)/3)/2);
yu1=yu1-h2*(yu2+h2*(yu3+h2*(yu4+h2*yu5/4)/3)/2);
yu2=yu2+h2*(yu3+h2*(yu4+h2*(yu5+h2*yu6/4)/3)/2);
r0=r0+h*(r1+h*(r2+h*(r3+h*r4/4)/3)/2);
ru0=ru0+h2*(ru1-h2*(ru2+h2*(ru3+h2*ru4/4)/3)/2);
x=x+h; x2=1-x; y(m+i+1)=y0; y(m-i-1)=yu0;
end
yy=yu0;

```

---

#### Test4

```

a=1; b=2; n=20; h=(b-a)/n; d=[1 0 0 0;0 1 0 0;0 0 1 0;0 0 0 1];
y00=1.8; y10=1.8; y20=1.95; w=9.6; c=[y00;y10;y20;w];
test4_4
f1=2*y00-y1+y10-2; f2=y2-2*y20+y00; f3=y1-y00-y0+3; f4=r0-w; f=[f1;f2;f3;f4];
disp('j      c1      c2      c3      c4      f1      f2      f3      f4 ')
fprintf('%2i %12.7f %12.7f %12.7f %12.7f %12.7f %12.7f %12.7f %12.7f\n',0,c',f)
for j=1:25
    b=linsolve(d,-f);
    c=b+c;
    y00=c(1); y10=c(2); y20=c(3); w=c(4);
test4_4
f1=2*y00-y1+y10-2; f2=y2-2*y20+y00; f3=y1-y00-y0+3; f4=r0-w; f=[f1;f2;f3;f4];
aa=f/(b'*b); d=d+(aa*b');
fprintf('%2i %12.7f %12.7f %12.7f %12.7f %12.7f %12.7f %12.7f %12.7f\n',j,c',f)
end
x1=1;
disp('i      y(x)      exact      err')
for i=1:n

```

```

x1=x1+h;
z=x1^2-1;

fprintf('%2i %12.7f%12.7f%12.7f\n',i,xx(i),z,z-xx(i))
end
% test4_4
x1=1; x2=1/x1; y0=y00; y1=y10; y2=y20; yu0=y00; yu1=y10; yu2=y20; r0=0; ru0=0;
for i=1:n
x12=x1^2; x13=x12*x1; x14=x13*x1; x15=x14*x1; x16=x15*x1;
x22=x2^2; x23=x22*x2; x24=x23*x2; x25=x24*x2; x26=x25*x2;
y3=y2^2-2*yu2-y1*y2+2*x12*yu1+w-28/3;
yu3=yu2^2-2*y2-yu1*yu2+2*x22*y1+w-28/3;
r1=y1^2; ru1=yu1^2;
y4=2*y2*y3+2*yu3/x12-y1*y3-y2^2-2*yu2+4*x1*yu1;
yu4=2*yu2*yu3+2*y3/x22-yu1*yu3-yu2^2-2*y2+4*x2*y1;
r2=2*y1*y2; ru2=2*yu1*yu2;
y5=2*y2*y4+2*y3^2-2*yu4/x14-4*yu3/x13-y1*y4-3*y2*y3+2*yu3/x12-
4*yu2/x1-4*yu1;
yu5=2*yu2*yu4+2*yu3^2-2*y4/x24-4*y3/x23-yu1*yu4-3*yu2*yu3+2*y3/x22-
4*y2/x2+4*y1;
r3=2*y1*y3+2*y2^2; ru3=2*yu1*yu3+2*yu2^2;
y6=2*y2*y5+6*y3*y4+2*yu5/x16+12*yu4/x15+12*yu3/x14-y1*y5-4*y2*y4-
3*y3^2-2*yu4/x14;
yu6=2*yu2*yu5+6*yu3*yu4+2*y5/x26+12*y4/x25+12*y3/x24-yu1*yu5-4*yu2*yu4-
3*yu3^2-2*y4/x24;
r4=2*y1*y4+6*y2*y3; ru4=2*yu1*yu4+6*yu2*yu3;
x0=x2; x1=x1+h; x2=1/x1; h2=x2-x0;
y0=y0+h*(y1+h*(y2+h*(y3+h*y4/4)/3)/2);
y1=y1+h*(y2+h*(y3+h*(y4+h*y5/4)/3)/2);
y2=y2+h*(y3+h*(y4+h*(y5+h*y6/4)/3)/2);
yu0=yu0+h2*(yu1+h2*(yu2+h2*(yu3+h2*yu4/4)/3)/2);

```

```

yu1=yu1+h2*(yu2+h2*(yu3+h2*(yu4+h2*yu5/4)/3)/2);
yu2=yu2+h2*(yu3+h2*(yu4+h2*(yu5+h2*yu6/4)/3)/2);
r0=r0+h*(r1+h*(r2+h*(r3+h*r4/4)/3)/2);
ru0=ru0+h2*(ru1+h2*(ru2+h2*(ru3+h2*ru4/4)/3)/2);
xx(i)=y0;
end

```

---

### Test5

```

p2=pi/2; p4=pi/4; a=0; b=p2; n=20; m=n/2; h=(b-a)/n; h2=-h; d=[1 0 0 0;0 1 0 0;0 0 1 0;0 0 0 1];
y00=1.5; y10=1; y20=-3; w=1; c=[y00;y10;y20;w];
test5_5
f1=yu1+3*y1+yu0; f2=yu2+y0+y2-p2; f3=yu2+y1+yu0+1; f4=ru0; f=[f1;f2;f3;f4];
disp('j      c1      c2      c3      c4      f1      f2      f3      f4      ')
fprintf('%2i %12.7f %12.7f %12.7f %12.7f %12.7f %12.7f %12.7f %12.7f\n',0,c,f)
for j=1:15
    b=linsolve(d,-f);
    c=b+c;
    y00=c (1); y10=c (2); y20=c(3); w=c(4);
test5_5
f1=yu1+3*y1+yu0; f2=yu2+y0+y2-p2; f3=yu2+y1+yu0+1; f4=ru0; f=[f1;f2;f3;f4];
aa=f/(b'*b); d=d+(aa*b');
fprintf('%2i %12.7f %12.7f %12.7f %12.7f %12.7f %12.7f %12.7f %12.7f\n',j,c,f)
end
x=0;
disp('i      y(x)      exact      err')
for i=1:n+1
    z=sin(2*x)+x;
    x=x+h;
    fprintf('%2i %12.7f %12.7f %12.7f\n',i,y(i),z,z-y(i))

```

end

**%test5\_5**

x=p4; x2=p2-x; y0=y00; y1=y10; y2=y20; yu0=y0; yu1=y1; yu2=y2; r0=w; ru0=w; y(m+1)=y0;

for i=1:m

c2=cos(2\*x); s2=sin(2\*x); c22=cos(2\*x2); s22=sin(2\*x2);

c4=cos(4\*x); s4=sin(4\*x); c44=cos(4\*x2); s44=sin(4\*x2);

y3=c2\*yu2/4-y0\*yu1/2-4\*y1+s4/8-x^3/3+4+r0;

yu3=c22\*y2/4-yu0\*y1/2-4\*yu1+s44/8-x2^3/3+4+ru0;

r1=y0^2; ru1=yu0^2;

y4=-c2\*yu3/4-s2\*yu2/2+y0\*yu2/2-y1\*yu1/2-4\*y2+c4/2-x^2+r1;

yu4=-c22\*y3/4-s22\*y2/2+yu0\*y2/2-yu1\*y1/2-4\*yu2+c44/2-x2^2+ru1;

r2=2\*y0\*y1; ru2=2\*yu0\*yu1;

y5=c2\*yu4/4+s2\*yu3-c2\*yu2-y0\*yu3/2+y1\*yu2-y2\*yu1/2-4\*y3-2\*s4-2\*x+r2;

yu5=c22\*y4/4+s22\*y3-c22\*y2-yu0\*y3/2+yu1\*y2-yu2\*y1/2-4\*yu3-2\*s44-2\*x2+ru2;

r3=2\*y0\*y2-2\*y1^2; ru3=2\*yu0\*yu2+2\*yu1^2;

y6=-c2\*yu5/4-3\*s2\*yu4/2+3\*c2\*yu3+2\*s2\*yu2+y0\*yu4/2-3\*y1\*yu3/2+3\*y2\*yu2/2-

y3\*yu1/2-4\*y4-8\*c4-2+r3;

yu6=-c22\*y5/4-3\*s22\*y4/2+3\*c22\*y3+2\*s22\*y2+yu0\*y4/2-3\*yu1\*y3/2+

3\*yu2\*y2/2-yu3\*y1/2-4\*yu4-8\*c44-2+ru3;

r4=2\*y0\*y3+6\*y1\*y2; ru4=2\*yu0\*yu3+6\*yu1\*yu2;

y0=y0+h\*(y1+h\*(y2+h\*(y3+h\*y4/4)/3)/2);

y1=y1+h\*(y2+h\*(y3+h\*(y4+h\*y5/4)/3)/2);

y2=y2-h\*(y3-h\*(y4-h\*(y5+h\*y6/4)/3)/2);

yu0=yu0+h2\*(yu1-h2\*(yu2-h2\*(yu3+h2\*yu4/4)/3)/2);

yu1=yu1-h2\*(yu2+h2\*(yu3+h2\*(yu4+h2\*yu5/4)/3)/2);

yu2=yu2+h2\*(yu3+h2\*(yu4+h2\*(yu5+h2\*yu6/4)/3)/2);

r0=r0+h\*(r1+h\*(r2+h\*(r3-h\*r4/4)/3)/2);

ru0=ru0-h2\*(ru1+h2\*(ru2+h2\*(ru3+h2\*ru4/4)/3)/2);

y(m+i+1)=y0; y(m-i+1)=yu0; x=x+h; x2=p2-x;

end